Calcul des déformations des ressorts coniques

Forces et couples concentrés - Modèle encastré - libre

Torseur des forces de cohésion

$$\mathbf{r_F}(r_F, \chi_F, \psi_F) := r_F \cdot \begin{pmatrix} \sin(\chi_F)\cdot\cos(\psi_F) \\ \sin(\chi_F)\cdot\sin(\psi_F) \\ \cos(\chi_F) \end{pmatrix} \qquad \mathbf{r_S}(\alpha') := \begin{pmatrix} x_0(\alpha') \\ y_0(\alpha') \\ z_0(\alpha') \end{pmatrix} \qquad \mathbf{r_v}(\alpha) := \begin{pmatrix} x_0(\alpha) \\ y_0(\alpha) \\ z_0(\alpha) \end{pmatrix}$$

Forces et couples concentrés en ψ_F

$$\mathbf{R_c}(\alpha') := \mathbf{F}$$

$$\mathbf{M_c}(\alpha') := \left[\mathbf{C} + \left(\mathbf{r_F}(r_F, \chi_F, \psi_F) - \mathbf{r_S}(\alpha')\right) \times \mathbf{F}\right]$$

Sollicitations

$$\mathbf{e'_1}(\alpha') := \left(-\cos(\beta(\alpha'))\cdot\sin(\alpha') \quad \cos(\beta(\alpha'))\cdot\cos(\alpha') \quad \sin(\beta(\alpha'))\right)^T \qquad \mathbf{e'_2}(\alpha') := \left(-\cos(\alpha') \quad -\sin(\alpha') \quad 0\right)^T$$

$$\mathbf{e'_3}(\alpha') := \left(\sin(\beta(\alpha'))\cdot\sin(\alpha') \quad -\sin(\beta(\alpha'))\cdot\cos(\alpha') \quad \cos(\beta(\alpha'))\right)^T$$

Traction-compression $\quad N_c(\alpha') := \mathbf{R_c}(\alpha')\cdot\mathbf{e'_1}(\alpha')$

Effort tranchant $\quad Q_2(\alpha') := \mathbf{R_c}(\alpha')\cdot\mathbf{e'_2}(\alpha')$

$Q_3(\alpha') := \mathbf{R_c}(\alpha')\cdot\mathbf{e'_3}(\alpha')$

Moment de torsion $\quad M_t(\alpha') := \mathbf{M_c}(\alpha')\cdot\mathbf{e'_1}(\alpha')$

Moments de flexion $\quad M_{f2}(\alpha') := \mathbf{M_c}(\alpha')\cdot\mathbf{e'_2}(\alpha')$

$M_{f3}(\alpha') := \mathbf{M_c}(\alpha')\cdot\mathbf{e'_3}(\alpha')$

Contraintes

k = limite d'élasticité en traction / limite d'élasticité en compression

M=1 point intérieur de la section droite sur l'axe O**e'**$_2$ M=3 point extérieur de la section droite sur l'axe O**e'**$_2$

M=2 point inférieur de la section droite sur l'axe O**e'**$_3$ M=4 point supérieur de la section droite sur l'axe O**e'**$_3$

$$\tau_{Mt}(\alpha', M) := \frac{M_t(\alpha')}{W_t}\cdot[(M=1) - 1\cdot(M=3)] + \frac{M_t(\alpha')}{W'_t}\cdot[(M=4) - 1\cdot(M=2)]$$

$$\tau_Q(\alpha', M) := \frac{Q_2(\alpha')}{S}\cdot[(M=2) + (M=4)] + \frac{Q_3(\alpha')}{S}\cdot[(M=1) + (M=3)] \qquad \text{(approximation)}$$

$$\sigma_{Mf}(\alpha', M) := \frac{M_{f3}(\alpha')}{W_{f3}}\cdot[(M=3) - 1\cdot(M=1)] + \frac{M_{f2}(\alpha')}{W_{f2}}\cdot[(M=2) - 1\cdot(M=4)]$$

$$\sigma_N(\alpha') := \frac{N_c(\alpha')}{S} \qquad \sigma_M(\alpha', M) := \sigma_{Mf}(\alpha', M) + \sigma_N(\alpha') \qquad \tau_M(\alpha', M) := \tau_{Mt}(\alpha', M) + \tau_Q(\alpha', M)$$

$$\sigma_{\acute{e}quiv}(k, \alpha', M) := \frac{1-k}{2}\cdot\left|\sigma_M(\alpha', M)\right| + \frac{1+k}{2}\cdot\sqrt{\sigma_M(\alpha', M)^2 + 4\cdot\tau_M(\alpha', M)^2}$$

Calcul des déplacements par les intégrales de Mohr

Calcul des déplacements linéiques d'un point du ressort

Force unitaire virtuelle $\mathbf{v}(\lambda,\gamma) := \left(cos(\lambda)\cdot sin(\gamma) \quad sin(\lambda)\cdot sin(\gamma) \quad cos(\gamma)\right)^T$

Sollicitations dues à la force unitaire

$$\mathbf{M_v}(\alpha,\lambda,\gamma,\alpha') := \left[\left(\mathbf{r_v}(\alpha) - \mathbf{r_S}(\alpha')\right)\times \mathbf{v}(\lambda,\gamma)\right]\cdot(\alpha' < \alpha) \qquad M_{tv}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot \mathbf{e'_1}(\alpha')$$

$$M_{fv2}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot \mathbf{e'_2}(\alpha') \qquad M_{fv3}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot \mathbf{e'_3}(\alpha')$$

Déplacement dans la direction de **v**

$$\delta_{tv}(\alpha,\lambda,\gamma) := \frac{1}{G\cdot J_t}\cdot\int_0^{\alpha} M_t(\alpha')\cdot M_{tv}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\delta_{fv2}(\alpha,\lambda,\gamma) := \frac{1}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{f2}(\alpha')\cdot M_{fv2}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\delta_{fv3}(\alpha,\lambda,\gamma) := \frac{1}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{f3}(\alpha')\cdot M_{fv3}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\delta_v(\alpha,\lambda,\gamma) := \delta_{tv}(\alpha,\lambda,\gamma) + \delta_{fv2}(\alpha,\lambda,\gamma) + \delta_{fv3}(\alpha,\lambda,\gamma)$$

Calcul des déplacements angulaires

Couple unitaire virtuel $\mathbf{cv}(\lambda_c,\gamma_c) := \left(cos(\lambda_c)\cdot sin(\gamma_c) \quad sin(\lambda_c)\cdot sin(\gamma_c) \quad cos(\gamma_c)\right)^T$

Sollicitations dues au couple unitaire

$$\mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{cv}(\lambda_c,\gamma_c)\cdot(\alpha' < \alpha)$$

$$M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot \mathbf{e'_1}(\alpha')$$

$$M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot \mathbf{e'_2}(\alpha') \qquad M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot \mathbf{e'_3}(\alpha')$$

Déplacement angulaire autour de l'axe défini par **cv**

$$\theta_{tcv}(\alpha,\lambda_c,\gamma_c) := \frac{1}{G\cdot J_t}\cdot\int_0^{\alpha} M_t(\alpha')\cdot M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\theta_{fcv2}(\alpha,\lambda_c,\gamma_c) := \frac{1}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{f2}(\alpha')\cdot M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\theta_{fcv3}(\alpha,\lambda_c,\gamma_c) := \frac{1}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{f3}(\alpha')\cdot M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{r_0(\alpha')}{cos(\beta(\alpha'))}\,d\alpha'$$

$$\theta_{cv}(\alpha,\lambda_c,\gamma_c) := \theta_{tcv}(\alpha,\lambda_c,\gamma_c) + \theta_{fcv2}(\alpha,\lambda_c,\gamma_c) + \theta_{fcv3}(\alpha,\lambda_c,\gamma_c)$$